# Probabilistic Reliability Analysis of Reinforced Concrete Beams in Fire Situation

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Abstract—The assessment of structures in fire situations involves design methodologies predominantly based on empirical studies of fire behavior and associated structural responses. Considering the uncertainties pertinent to the issue, the reliability of reinforced concrete structures in fire situations must be investigated to ascertain the degree of safety that reflects the physical and mathematical models utilized in calculations and the statistical models used in quantifying uncertainties. In this context, this work aims to determine, through a computational thermal analysis based on the Finite Element Method, the load-carrying capacity of a group of typical reinforced concrete beams in residential buildings subjected to simple bending and exposed to fire ISO 834. Subsequently, the Reliability Indices will be established using the FORM method, with a parametric study conducted to assess the influence of the critical random variables related to strength and design load. The results show a significant dispersion in the obtained Reliability Indices values, with the lowest values corresponding to action combinations where the accidental load is proportionally equal to or smaller than the permanent load. This reliability reduction suggests that the current partial safety coefficients employed in the exceptional ultimate combination of actions for the fire situation, as specified by Brazilian standards, do not ensure uniformity in the Reliability Indices of the beams. Consequently, a review based on a probabilistic analysis of the partial safety coefficients is advisable.

Keywords—Fire, Finite Element Method, Structural Reliability.

## I. INTRODUCTION

A series of historical fires in Brazil, dating back to the 1970s, have profoundly impacted the revision of Brazilian fire prevention and protection norms. Before the early 1970s, Brazil's fire prevention and protection regulations were primarily embedded within municipal building codes, confined to prescribing recommendations for the width of emergency exits in buildings.

Only in December 2004, the publication of [1] brought the requirements for the design of reinforced concrete structures, requiring the use of appropriate calculation methods to evaluate the resistant capacity of the structures as a function of the required time fire resistance demanded by [2].

According [3], the standards [1], as well as other Brazilian standards for the design of structures, are grounded in the Ultimate Limit State Method. This approach indirectly accounts for uncertainties intrinsic to the structure, material properties, and external forces by incorporating partial safety factors to ensure satisfactory user safety. However, considering the advances in studies in the area, it is recognized that the optimal strategy for assessing structural safety lies in a probabilistic evaluation. Mauro de Vasconcello Real Programa de Pós Graduação em Modelagem Computacional Universidade Federal do Rio Grande - FURG Rio Grande, Brazil mauroreal@furg.br

Designing structures under fire conditions presents a formidable challenge for engineers. This challenge occurs primarily because design methodologies are predominantly based on empirical studies of fire behavior and associated structural responses. Given the multitude of uncertainties intrinsic to this issue (such as the altered mechanical traits of materials at elevated temperatures, dimensional variations, the application of mathematical models, and more), it becomes evident that the reliability of reinforced concrete structures in fire situations must be investigated.

Regarding the bibliographies in this domain, it has been observed that several studies have already been conducted to evaluate the reliability of structural elements at ambient temperature. One notable example is the work by [3], which presented an investigation into the safety of beams subjected to simple bending in reinforced concrete, steel, and mixed material and dimensioned according to Brazilian standards.

Reference [4] presented a study on the safety of reinforced concrete beams subjected to bending moments. Using the Reliability Theory, they designed the beams according to Brazilian standards to evaluate the uniformity in the safety of structures for different positions of the neutral axis and ratio of loads. Reference [5] and [6] evaluated the reliability of reinforced concrete beams at ambient temperature in different design situations and designed according to [7].

In the context of structures under fire conditions, which is the focal point of this study, notable references from international literature include the research conducted by [8] on the reliability analysis of reinforced concrete columns during fire exposure, as well as the work by [9] concerning the reliability analysis of reinforced concrete beams subjected to fire conditions.

There are few national works on the reliability of structures in fire situations, with [10], who carried out the structural reliability analysis of steel elements at high temperatures. Recently, [11] assessed the reliability of cross-sections of simply supported reinforced concrete beams in a fire situation using the simplified method proposed by the [1] standard. The influence of concrete cover, load ratio, and the number of heated faces of the structure were considered in the analysis. It was found that an increase in concrete cover enhances reliability, while conversely, a higher proportion of variable load to total load leads to a decrease in failure probability. Regarding a sensitivity analysis, fire temperature and concrete cover were the most impactful variables.

Thus, as a provisional speculative proposition for the investigative starting point of this work, numerically estimating the safety level of the Brazilian standard for the design of reinforced concrete structures in fire situations through a reliability analysis of a selection of usual beams of residential buildings, subjected to the Ultimate Limit State of simple bending and exposed to fire.

Through a two-dimensional numerical analysis based on the Finite Element Method, implemented through the ANSYS Mechanical APDL software, the distribution of temperatures in the cross-section of the beams subjected to the action of the ISO 834 fire curve will be determined. Subsequently, the resistance capacity of the structures is determined through a Simplified Verification Method, according to [1]. The verification of the fire action model is done through a comparison with the values obtained through the Wickström Method and the verification of the Resistant Moment of calculation in a fire situation (Mr,fi) with o Isotherm Method 500°C, according to the [12].

Therefore, a statistical investigation of the basic random variables of resistance and loading present in the failure function of the structural elements is done. The level 3 reliability analysis is carried out through the First Order Reliability Method (FORM) of the structural elements in the Ultimate Limit State to determine the Reliability Index ( $\beta$ ) achieved in the design of beams in a fire situation, considering various dimensions of cross-section, steel areas, type of aggregate, value of compressive characteristic strength concrete and load ratios.

In the end, the graphs with the results of the Reliability Index, considering the variability of each analyzed parameter, are presented and discussed.

#### II. METHODOLOGY

#### A. Characteristics of the analyzed beams

The reinforced concrete beams analyzed in this study, designed for the ultimate limit state of simple bending according to [7], have a length of 6 m and a rectangular cross-section of 20x50 cm and 25x70 cm, are subject to uniformly distributed permanent and accidental loads, and are assumed to belong to a residential building 25 m high. Two values of characteristic strength of concrete were assumed (25 MPa and 35 MPa), and two types of aggregates (limestone and siliceous). The yield strength of the longitudinal reinforcement steel is assumed to be 500 MPa, and the environmental aggressiveness class is CAA II.

 
 TABLE I.
 GEOMETRIC AND PHYSICAL CHARACTERISTICS OF THE CONCRETE SECTION.

l (cm)	b (cm)	h (cm)	Ac (cm <sup>2</sup> )	fck (MPa)	Aggregates
600	20	50	1000	25/35	siliceous/limestone
600	25	70	1750	25/35	siliceous/limestone

TABLE II. CHARACTERISTICS OF LONGITUDINAL REINFORCEMENT.

Number of bars	φ (mm)	As (cm <sup>2</sup> )	d' (cm)
4	8.0	2.01	3.90
4	10.0	3.14	4.00
4	12.5	4.91	4.13
3	16.0	6.03	4.30
3	20.0	9.42	4.50

Table I presents the geometric and physical characteristics of the concrete section of a group of structures typically found in residential buildings, and Table II shows the data related to the longitudinal reinforcement of the beams, which were divided into two groups respecting the minimum reinforcement rate required by [7].

#### B. Computational termal analysis

A finite-element computational tool was developed to calculate the structural response of concrete members during fire iteratively and to enable a full-probabilistic analysis of typical concrete elements subjected to bending.

The ANSYS Mechanical APDL software was adopted as a thermal analysis tool, based on the Finite Element Method, through the language known as ANSYS Parametric Design Language, which has resources for declaring variables, executing loops, making logics decisions, assignment of values, mathematical operators and parametric functions.

In general, the computational modeling performed in the software is divided into three main stages, which will be detailed in the following items: pre-processor, solution (processing), and post-processor.

### B.1. Pre-processor

From the modeling of the concrete mass, the thermal parameters of the material are assigned, which vary with temperature, following the recommendations of [1]. The specific heat was considered for 0% weight humidity, and the finite element used for mesh discretization was the PLANE55 (2-D Thermal Solid).

The computational domain for the application of the Finite Element Method was discretized by a square mesh with a side length of 0.50 cm, defined through a mesh convergence test, where the results were considered satisfactory when the relative percentage difference between the internal temperature in the concrete at a specific point, for two consecutive meshes, was less than 0.5%.

#### **B.2.** Processing

For the processing of the structure, the type of analysis was defined considering the material properties as variable over a given period, that is, a transient thermal analysis. A time step of 1 second was adopted, as simulations with this time step yielded satisfactory results with an average processing time of less than 5 minutes.

For defining the boundary and initial conditions of the analysis, the effects of convection and heat radiation on the cross-sectional beams' external lateral and bottom walls were considered, as shown in Fig. 1.

The surface temperature elevation was determined using the Analytical Method of Wickström. Meanwhile, the temperature variation due to conduction within the beam was solved using Fourier's Law and the Finite Element Method.

The upper face was considered adiabatic as a simplified assumption for cases where the beam is under a slab. The thermal action of the standard ISO 834 fire curve was considered for 90 minutes on the structure, equivalent to the Required Fire Resistance Time (RFRT) as required by [2]. The initial temperature of the structure was assumed to be 20°C.



Fig. 1. Boundary conditions for computational thermal analysis.

In the computational modeling of the cross-sectional section, the steel bars present in the concrete mass were disregarded because the thermal resistance of steel is much lower than that of concrete. Therefore, it is assumed that the steel will instantly absorb the temperature of the concrete, and only the properties of the concrete sensitive to thermal action are considered.

All the internal forces resulting from imposed deformations were neglected due to their small magnitude and significant plastic deformations during a fire situation. Thus, the fire action only results in the reduction of material strength and the capacity of the structural element.

According to [13], spalling was disregarded because in concretes with conventional strength (fck  $\leq$  50 MPa), this phenomenon rarely occurs. Therefore, it is uneconomical to try solutions to prevent it.

#### B.3. Post Processor

The outputs obtained after the thermal analysis are analyzed in the post-processing stage. These results include the temperatures at the nodes of the finite element mesh, from which the material strength reduction factors are determined:  $kc,\theta i$  for each node of the compressed concrete finite elements, and  $ks,\theta i$  for each steel bar (considering the bar's temperature equal to the concrete's temperature on its axis, obtained through linear interpolation of normative values).

Considering the equation for solving simple bending problems based on the equilibrium of the resulting forces in the tensioned steel bars and the compressed concrete stress block, it is possible to determine the lever arm (Z,fi) relative to the center of this stress block. This lever arm is the main unknown for calculating the resistant moment in a fire situation, as illustrated in Fig. 2, where Fsd,fi is the calculated resultant force in the reinforcement in a fire situation, and Fcd,fi is the calculated resultant force in the compressed concrete block in a fire situation.

The fundamental deterministic model for calculating temperature-dependent flexural strength was developed in Microsoft Excel software, considering exposure to the ISO 834 fire curve for the required fire resistance duration.



Fig. 2. Balance of simple bending forces in a fire situation.

#### C. Reliability analysis

According to [14], using the FORM method, the reliability analysis of the beams in this study begins with defining the limit state function that governs the problem. Then, the main random variables involved in the design are described.

#### C.1. Ultimate Limit State Function

The reliability analysis of the beams in a fire situation in the ultimate limit state for simple bending is given by (1):

$$g(X) = R(Y) - S(Z) \tag{1}$$

where X is the vector of random variables in the problem; R(Y) is the function of random variables representing the structural element's resistance moment; and S(Z) is the function of random variables representing the external moment.

In terms of the variables contained in the vectors Y and Z, the ultimate limit state function can be rewritten as (2):

$$g(X) = Em, r(As*ks*fy*(d-2))$$

$$0,5((As*ks*fy)/(kc*fc*b))) - Em, s(Fg(l^2/8) + Fq(l^2/8))$$
(2)

where Em, r is the strength model error, As is the longitudinal reinforcement area, ks is the steel resistance reduction factor, fy is the yield strength of the reinforcement, d is the effective height of the beam, kc is the concrete resistance reduction factor, fc is the compressive strength of concrete, b is the beam width, Em, s is the load model error, Fg is the dead load, l is the calculated length of the beam and Fq is the live load.

The values used for the steel and concrete resistance reduction coefficients were obtained from the thermal analysis conducted in ANSYS Mechanical APDL, considering the ISO 834 fire exposure for 90 minutes. To determine the steel resistance reduction factor (ks), the average value obtained for ks, $\theta$ i for each steel bar was considered. For the determination of the concrete resistance reduction factor (kc), the calculated resultant force value in the compressed concrete block under fire conditions was used. The parameters related to the random variables of resistance and load are presented below.

#### C.2. Radom variables

The main random resistance variables of the materials used in this study were obtained from the research conducted by [15], which serves as a reference for getting these parameters according to the reality of Brazilian construction projects.

As for the geometric characteristics of the cross-sectional area, the guidelines of [7] and the distribution proposed by [16] were adopted for the base (b) and effective height (d) random variables related to the dimensions of the cross-section of reinforced concrete beams.

The random variable associated with the resistance model error (E(m,r)) expresses the difference between the actual behavior of a structural element and the behavior predicted for it based on the calculation model employed in its design. The adopted value was derived from the study by [17], which investigated flexural models of reinforced concrete beams, including experimental results for beams with different heights, reinforcement ratios, and concrete strengths.

Reference [18] found that designers underestimate the self-weight of structures on average by 5% and make errors with a coefficient of variation of 10%. The variable describing the error about the actual self-weight follows a normal distribution so that it can be written as Fg-N(1,05Fgk,10%), where Fgk is the characteristic value. Similar results were obtained by [15] in a limited experiment with Brazilian designers.

Reference [19] demonstrated that the probability of a fire coinciding with peak values of variable loads is insignificant, and it is likely that a structure is loaded with only a fraction of variable load when a fire occurs. Therefore, based on [20], it is advisable to use the combination of variable loads at an arbitrary point in time for reliability analysis under fire conditions. This hypothesis is also consistent with the recommendation of [22] and [14].

Table III summarizes the statistical parameters of the basic random variables for resistance and load present in the Ultimate Limit State function considered in this study (mean, coefficient of variation, and probability distribution type), along with the bibliographic reference.

 
 TABLE III.
 STATISTICS OF RANDOM VARIABLES FOR RESISTANCE AND LOAD.

Variable	Distributio n	Average	Coefficient of variation	Source
fc (25 MPa)	Normal	1.25 fck	0.17	[15]
fc (35 MPa)	Normal	1.19 fck	0.13	[15]
fy	Normal	1.22 fyk	0.04	[15]
b	Normal	1.01 bn	0.04	[9]
d	Normal	0.99 dn	0.04	[9]
Em,r	Normal	1.02	0.06	[17]
Fg	Normal	1.05 Fgk	0.10	[18]
Fq	Gamma	0.25 Fqk	0.55	[18]
Em,s	Lognormal	1.00	0.05	[14]

Table III summarizes the statistical parameters of the basic random resistance and load variables present in the Ultimate Limit State function considered in this study (mean, coefficient of variation, and type of probability distribution), along with the bibliographic reference.

## C.3. Reliability index

The methodology for calculating the reliability index through the First-Order Reliability Method (FORM) is based on the Hasofer-Lind model proposed by [22]. In [23], this methodology was developed using the Solver tool integrated with Microsoft Excel software, utilizing the Visual Basic for Applications (VBA) programming language inherent to this tool.

Applying the FORM method involves constructing a joint probability distribution function and transforming it into a reduced multivariate normal distribution. The spreadsheet developed by [23] is user-friendly and built on an optimization routine. Input data for the spreadsheet includes the mean and standard deviation values of the random variables for resistance and load involved in the Limit State Function describing the problem and the distribution type for each variable, as presented in Table III.

By subjecting the Limit State Function describing the problem to the constraint g(X)=0, the Solver tool calculates the design point values and consequently  $\beta$  by automatically adjusting the design point values in the standardized normal space of each random variable. The Reliability Index value can be established after determining the minimum distance between the failure point and the system's origin.

## III. RESULTS AND DISCUSSIONS

#### A. Temperature verification

Initially, the temperatures on the concrete surface under the effect of the ISO 834 standard fire were examined over time. The verification involves a comparison of the temperature on the structure's surface according to Wickström Method and the modeling in ANSYS Mechanical APDL for different fire durations, considering beams with a section of 20x50 cm. The results are presented in Table IV.

Duration of Standard	Wickström Equation	ANSYS	Difference	
ISO 834 Fire (min)		(node 10) V20x50	(°C)	(%)
30	746	755.05	8.69	1.2
60	887	890.38	3.27	0.4
90	963	964.5	1.88	0.2
120	1014	1015.2	1.27	0.1

TABLE IV. VERIFICATION OF TEMPERATURE ON THE CONCRETE SURFACE (°C).

A comparison was also sought with the results obtained using the Wickström Method, at a point located 5 cm away from the bottom face and 5 cm from the left side face of the section, against the values obtained in the ANSYS Mechanical APDL modeling, as shown in Table V.

By observing Tables IV and V, it is noted that, although there is a more significant difference between the temperature data inside the concrete section compared to the surface temperature variation, the results of verifying the model created in ANSYS Mechanical APDL with the Wickström equation are satisfactory, as this difference is less than 30 °C or 7%.

Duration of Standard	Wickström Equation	ANSYS	Difference	
ISO 834 Fire (min)		(node 10) V20x50	(°C)	(%)
30	201.06	200.90	-0.16	0.1
60	416.57	387.79	-28.78	6.9
90	550.19	523.35	-26.84	4.9
120	646.06	627.87	-18.19	2.8

TABLE V. VERIFICATION OF TEMPERATURE INSIDE THE CONCRETE (°C).

# B. Verification of Calculated Moment Resistance under Fire Conditions

The verification of calculated moment resistance under fire conditions, obtained from the temperature profiles resulting from the modeling in ANSYS Mechanical APDL for a fire exposure time of 90 minutes (RFRT), was compared with the values obtained through the Isotherm 500°C Method of [12], using the same temperatures and those obtained through the Wickström Method, as shown in Table VI and Table VII.

 TABLE VI.
 VERIFICATION OF CALCULATED MOMENT RESISTANCE

 UNDER FIRE CONDITIONS FOR BEAMS IN GROUP I.

	Wickström Temperature	ANSYS Temperature		
Beams	Isotherm 500°C	Isotherm 500°C	Simplified Method ANSYS	
V101	24.33	26.16	26.15	
V102	39.27	42.05	42.04	
V103	63.13	66.88	66.85	
V104	76.90	79.85	79.81	
V105	123.06	128.89	128.78	
V106	24.33	26.16	26.20	
V107	39.27	42.05	42.17	
V108	63.13	66.88	67.21	
V109	76.90	79.85	80.33	
V110	123.06	128.89	130.28	
V111	24.45	26.30	26.29	
V112	39.59	42.42	42.41	
V113	64.00	67.86	67.84	
V114	78.22	81.28	81.26	
V115	126.79	133.03	132.94	
V116	24.45	26.30	26.33	
V117	39.59	42.42	42.51	
V118	64.00	67.86	68.09	
V119	78.22	81.28	81.63	
V120	126.79	133.03	134.02	

As it is shown by the values presented above, the three values obtained in the verification of calculated moment resistance under fire conditions for the two groups of analyzed beams are very close, with a maximum difference of 8% when

comparing temperatures in the structure according to the Wickström Method.

 TABLE VII.
 VERIFICATION OF CALCULATED MOMENT RESISTANCE

 UNDER FIRE CONDITIONS FOR BEAMS IN GROUP II.

	Wickström Temperature	ANSYS Temperature		
Beams	Isotherm 500°C	Isotherm 500°C	Simplified Method ANSYS	
V201	52.79	66.48	66.51	
V202	84.69	106.64	106.70	
V203	118.74	126.70	126.79	
V204	190.78	208.82	209.09	
V205	52.79	66.48	66.58	
V206	84.69	106.64	106.91	
V207	118.74	126.70	127.09	
V208	190.78	208.82	209.94	
V209	52.99	66.80	66.82	
V210	85.22	107.48	107.53	
V211	119.80	127.91	127.98	
V212	193.65	212.29	212.49	
V213	52.99	66.80	66.87	
V214	85.22	107.48	107.68	
V215	119.80	127.91	128.19	
V216	193.65	212.29	213.09	

Considering the same thermal distribution, the calculated moment resistance under fire conditions obtained using the proposed Simplified Method yields results similar to those obtained by the Isotherm 500°C Method, which is internationally recognized.

# C. Reliability analisys

In building design, it is essential to consider different ratios between variable and permanent load values. According to [14], reinforced concrete buildings typically have values in the range  $0.5 \le r \le 1.5$ .

For reliability analyses, assumed values for the load ratio were r = 0.25, 0.5, 1.0, 1.5 and 2.0. Therefore, two beam cross-sections were analyzed under five loading conditions, considering different longitudinal reinforcement areas, two types of aggregates, and two values of fck, resulting in a total of 180 scenarios.

The Reliability Indices obtained for beams with fck=25MPa and siliceous aggregate are shown in Fig. 3 and Fig. 4. Fig. 5 and Fig. 6 present the results for  $\beta$  considering the same concrete compressive strength value and varying the aggregate type, this time limestone.

It is observed that, regardless of the type of aggregate, as the ratio of variable loading to permanent loading increases, there is an increase in the value of the reliability index, as demonstrated in the works of [3], [8], and [11].

It is also observed that the section height directly influences the value of  $\beta$ . Beams with greater height exhibit higher reliability indices, even when their reinforcement ratios are very close in value.



Fig. 3. Reliability Index for Beams with fck=25MPa, h=50 cm, and Siliceous Aggregate.



Fig. 4. Reliability Index for Beams with fck=25MPa, h=70 cm, and Limestone Aggregate.



Fig. 5. Reliability Index for Beams with fck=25MPa, h=50 cm, and Limestone Aggregate.



Fig. 6. Reliability Index for Beams with fck=25MPa, h=70 cm, and Limestone Aggregate.

Considering the same analyzed characteristics, but now for beams with fck=35MPa, as shown in Fig. 7 to Fig. 10, it is observed that the increase in concrete compressive strength has an insignificant influence on the values obtained for the Reliability Index.



Fig. 7. Reliability Index for Beams with fck=35MPa, h=50 cm, and Siliceous Aggregate.



Fig. 8. Reliability Index for Beams with fck=35MPa, h=70 cm, and Siliceous Aggregate.



Fig. 9. Reliability Index for Beams with fck=35MPa, h=50 cm, and Limestone Aggregate.



Fig. 10. Reliability Index for Beams with fck=35MPa, h=70 cm, and Limestone Aggregate.

# **IV. CONCLUSIONS**

A group of typical reinforced concrete beams from residential buildings, subjected to the ISO 834 fire exposure for the required fire resistance duration stipulated by standards, had their material mechanical properties at elevated temperatures determined through transient thermal modeling carried out in ANSYS Mechanical APDL software, based on the Finite Element Method. The verification of temperatures reached in the cross-sectional area of the structures and the values of the resisting capacities obtained through a Simplified Method was conducted using well-established methods found in international literature, and the results were satisfactory.

Reliability analysis based on the First Order Reliability Method (FORM) assessed critical random variables related to strength and loadings in the Ultimate Limit State function of simple bending for structures under fire conditions, considering the context of Brazilian construction practices.

The obtained values for the Reliability Indices of the beams exhibit trends based on the analyzed parameters, irrespective of the type of aggregate used. As the ratio of variable loading to permanent loading increases, the values of  $\beta$  also increase. Additionally, it is observed that the section height directly influences the  $\beta$  values, as beams with greater height yield higher Reliability Indices, even when their reinforcement ratios are very close. The increase in concrete compressive strength has an insignificant influence on the obtained Reliability Index values.

In general, there is a significant dispersion in the obtained values for Reliability Indices, with the lowest values corresponding to combinations of actions where the accidental load is proportionally equal to or smaller than the permanent load. This dispersion indicates that the current partial safety coefficients used in the exceptional ultimate combination of actions for the fire situation adopted by [1] do not ensure uniformity of  $\beta$ .

Based on the obtained results, it is concluded that the partial safety coefficients used in the Brazilian standard for the design of concrete structures under fire conditions lead to a significant variation in the safety levels of the structures. A review based on a probabilistic analysis of the partial coefficients could be advisable. However, this recommendation depends on a further investigation initiated in this study, as the obtained results do not encompass all design scenarios covered by this standard.

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